Deadline Differentiated Dynamic EV Charging Price Menu Design

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***Abstract*—The increasing number of electric vehicles (EVs) on the road brings both opportunities and challenges to the power system. For the EV charging stations (EVCSs), it is often difficult to conduct effective operations due to the incomplete information in EVs' departure times and the opacity of their preference information. To tackle this challenge, we seek to design the optimal deadline differentiated dynamic price menu that offers multiple choice-pairs of deadlines and charging prices. We prove that such price menus can incentivize EVs to truthfully reveal their departure time. We then analyze the properties of the optimal price menu with complete EV information, i.e., social optimality and first-degree price discrimination. For the incomplete information case, we first design a systematic method to estimate the utility and demand information for a large population of EVs based on EV behavior data. Then, we employ mixed-integer quadratic programming for the efficient optimal price menu design. The numerical study based on field data in California verifies the remarkable performance of our designed price menu.**

***Index Terms*—EV charging, price design, behavior analysis.**

NOMENCLATURE

Acronyms

EV	Electric Vehicle
EVCS	Electric Vehicle Charging Station
EDT	Earliest Departure Time

Sets

$\mathcal{M}$	Price menu
$\mathcal{M}_i$	Customized price menu for EV i
$\mathcal{M}_t$	Uniform price menu at t
$\mathcal{U}_i$	EV i's temporal utility
$\hat{\mathcal{S}}_t$	Clustered EV information set at t
$\hat{\mathcal{U}}_i^t$	Temporal utility set for EV pattern i at t

Parameters

T^{base} ($T_{t,i}^{base}$)	Minimal charging time (of EV pattern i at t)
T^{extend}	Contract extend period
T^{delay} (T_i^{delay})	EV's (EV i's) delay tolerance
ξ	Charging efficiency
$p^{\max}$	Maximal charging rate
P_0	Conventional unit charging price
C_{delay}	Unit overstay fee
C_{conv}	Conventional overall charging cost
H	Price menu length
D (D_i)	EV's (EV i's) charging demand
$\hat{D}_{t,i}$	Estimation for the demand of EV pattern i at t
$D_{t,i}$	The charging demand of EV i at t
c_t	Unit real time electricity price at t
t_i	EV i's arrival time
R_i	The revenue of EVCS by serving EV i
α_i, β_i	The parameters of EV i's utility function
I_i	EV i's average time value
ST_i	EV i's spatial-temporal trajectory entropy
ST_i^S	EV i's spatial trajectory entropy
ST_i^T	EV i's temporal trajectory entropy
W_i	The number of visited places of EV i
$D_{i,j}$	The travel distance for EV i to visit the next place from the jth place
$T_{i,j}$	The stay duration in j^{th} visited place of EV i
N_c^t	The number of target clusters at t
$\hat{\alpha}_{t,i}$ ($\hat{\beta}_{t,i}$)	Estimation for α (β) of EV pattern i at t
$\hat{\sigma}_{t,i}$	Estimation for proportion of EV pattern i at t
$\hat{U}_{i,k}^t$	Estimated utility for EV pattern i at t with delay tolerance k
θ_t	The social welfare allocation ratio at t
δ_t	The revenue transfer fee at t
$U_{t,conv}^{EVCS}$	The revenue for EVCS at t under conventional pricing scheme
$U_{t,conv}^{EV}$	The revenue for EV population at t under conventional pricing scheme

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$U_{i,k}^{t}$ The temporal utility of EV i at t with contract extend period k

Variables

P_k ($P_{t,k}$) Unit charging price with contract extend period k (at t)
C_{menu} Menu-based overall charging cost
$U_{i,k}$ EV i's temporal utility with delay tolerance k
Q_i EV i's overall utility
$P_{i,k}$ Unit charging price of the kth option of price menu $\mathcal{M}_i$
k_i^* EV i's optimal price menu choice
$k_{t,i}^*$ EV pattern i's optimal menu choice at t
$E(t, k, D)$ The minimal energy purchase cost at time t to satisfy an EV with menu choice k and charging demand D
$\hat{B}(t, i, k)$ The minimal energy purchase cost to satisfy EV pattern i's charging demand at t with contract extend period k
$B(t, i, k)$ The minimal energy purchase cost to satisfy the demand of EV i in population at t with contract extend period k
$a_{t,i,k}$ The binary indicator for whether EV pattern i arriving at t chooses the kth service
$U_{t,menu}^{EVCS}$ The revenue for EVCS at t by serving EV population with menu-based scheme
$U_{t,menu}^{EV}$ The utility for EV population at t under menu-based pricing scheme
SW_t The increased social welfare introduced by EVs' flexibility at t

I. Introduction

The combat with climate change warrants joint efforts from all sectors, ranging from the transportation sector to the electricity sector. With the global efforts in the electrification of the transportation sector, the last decade witnessed a dramatic increase in the number of electric vehicles (EVs) on the road, the number of EV charging stations (EVCSs), and the capacity of the EV charging market. In this paper, we focus on the EVCS operation issue.

Specifically, facing real-time electricity prices, the EVCS seeks to incentivize EVs to truthfully reveal their departure times. With such information, the EVCS can flexibly adjust the charging schedule to serve the EVs' charging demand at the minimal electricity purchase cost. However, the departure time is private information, which complicates the effective EVCS operation. Hence, it is crucial to design a proper mechanism to enable EV information revealing.

In order to tackle the challenge, we design a deadline differentiated dynamic charging price menu which includes multiple pairs of deadlines and corresponding charging prices. We prove that rational EVs will truthfully reveal their departure times when facing the price menu. Based on this property, we further design the optimal price menu for the cases with complete and incomplete information, respectively.

A. Related Works

We identify two major streams of closely related works. The first research stream is the investigation of EV charging price, and the second focuses on menu-based price design.

EV charging price design is at the core of EV's integration into the grid. Recent works concentrate on charging price design to maximize the profits of EVCSs. For example, Wang *et al.* propose a framework to conduct joint pricing for EV admission control and charging scheduling in [1]. Cui *et al.* design the optimal charging price for multiple EVCSs considering the inter-dependency between transportation network and power network in [2]. Mao *et al.* propose a vehicle-to-grid pricing policy combining system load condition, maximum charging limit, and residential electricity price in [3]. The uncertainty in the system warrants dynamic charging pricing. Luo *et al.* propose an approach to designing stochastic dynamic price for EVCSs dealing with charging demand volatility and renewable energy generation [4]. Soares *et al.* propose a dynamic pricing and day-ahead energy resource scheduling for EVs through stochastic optimization in [5]. Lu *et al.* characterize the equilibrium charging price considering the competitions among multiple EVCSs in [6], [7]. In addition, various machine learning methods have been applied to EV charging pricing recently, such as deep reinforcement learning [8], the multi-agent deep reinforcement learning [9].

However, most of these works focus on setting a single charging price at each time and do not consider providing EVs with multiple differentiated service options. Therefore, much can be improved for the EVCSs to better utilize the EVs' charging flexibility, which is our focus.

We are not the first to consider differentiated EV charging services. Tan and Wang propose a reliability-differentiated charging pricing mechanism to enhance the reliability of the distribution system in [10]. Moradipari and Alizadeh investigate pricing priority and charging demand-differentiated services for EVs offered by EVCS to maximize social welfare in [11]. As for pricing deadline-differentiated services, Bitar *et al.* propose a deadline-differentiated pricing approach for deferrable electric power services, which can reduce charging fees with increasing charging flexibility [12], [13]. However, these works only consider homogeneous EV utility function, without capturing the heterogeneous preferences among a large EV group. Ghosh and Aggarwal consider the menu-based EV charging pricing customized for each single EV in [14]. Based on this work, Ghosh and Aggarwal further extend their work to vehicle-to-grid charging scenarios in [15]. Salah and Flath design a uniform price menu for EVs with local PV generation in [16]. Zeng *et al.* design the price for both flexible and inflexible charging services, which incorporates EV behaviors in [17].

The major limitation of this research line is that they mainly consider the complete information case. However, in practice, the incomplete information about EVs will bring a large burden to the price menu calculation. The time-invariant price menu is also not customized and often suboptimal.

In contrast, we consider the scenario that the EVCS participates in the real-time electricity market and charges the EVs with an overstay fee, which is compatible with the overstay fee-based pricing used by most EVCSs. To tackle the problem caused by the incomplete EV information, we propose a systematic method to estimate the distribution of EV population's utility and demand based on large EV behavior data. Further, to tackle the computational intractability of large-scale EV population optimization, and enable dynamic price menu design for EVs on different days and at different times, we utilize the K-means clustering algorithm to extract the time-variant EV information patterns. Based on the estimated EV information, we design an efficient algorithm to construct the optimal price menu.

B. Our Contributions

We target to design the price menu such that the EVCS can provide customized services with different deadlines in both complete and incomplete information scenarios. Our contributions can be summarized as follows:

- *Price Menu Design with Complete Information*: We propose a constructive approach to design the optimal price menu yielding the maximal EVCS revenue. We prove its key properties, including social optimality and first-degree price discrimination.
- *EV Information Estimation*: We design a systematic and scalable method to dynamically estimate the distribution of EV population's utility and demand based on EV behavior data. We also utilize the K-means clustering algorithm to extract the time-variant EV information patterns.
- *Price Menu Design with Incomplete Information*: Based on the estimated EV information, we provide an efficient algorithm to solve the optimal price menu design, which transforms the original intractable bi-level mixed-integer optimization into a tractable mixed-integer quadratic programming (MIQP).
- *Extended Price Menu Design*: To enable a more flexible price menu design, we introduce the notion of revenue transfer fee to dynamically allocate social welfare between EVCS and EVs. Further, we demonstrate the scalability of the price menu design framework and extend it to design customized price menus for EVs with different charging demands.

Specifically, the logical flow of this paper is illustrated in Fig. 1, and our work is organized as follows: Section II introduces our system model, including the charging price menu, the EV choice model, and the EVCS model, which allow us to mathematically characterize the key challenge in price menu design, truthful departure time revealing. To tackle this issue, in Section III, we first design and analyze the optimal price menu with complete information for more theoretical insights. Based on such insights, we further our analysis to the practical and challenging incomplete information case. We estimate the EV population's utility and demand distribution based on EV behavior data in Section IV. These estimated parameters allow us to investigate the optimal price menu in Section V, where we also transform the original intractable mixed-integer bi-level optimization into a tractable

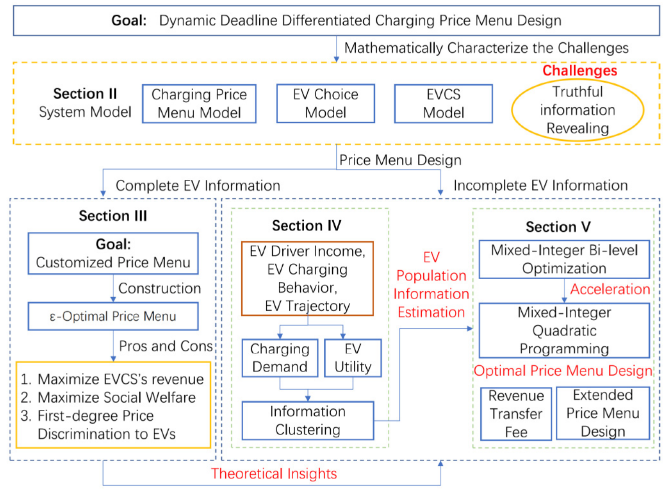


Fig. 1. Structural Diagram.

MIQP. In addition, we propose the advanced design for the price menu, including the revenue transfer fee and demand-differentiated price menu design. We numerically verify the performance of our proposed price menu design framework in Section VI. Finally, Section VII concludes our work and suggests interesting future directions. All the necessary proofs are deferred to the Appendix.

II. System Model

Consider the EV charging coordination for an EVCS. The EVs arrive at the EVCS in real-time to charge themselves. Before the charging service, the EVCS offers a charging price menu (possibly time-varying) to the EVs, and then provides the service according to their choices. The goal of the price menu is to better exploit the EVs' flexibility. In this section, we sequentially introduce the charging price menu model, the EV choice model, and the EVCS model.

Specifically, in subsequent analysis, we discretize the time into slots when considering EV's arrival, departure, and the corresponding price design. To improve the readability, we first introduce the following definitions:

Definition 1: Earliest Departure Time (EDT): the earliest time to satisfy the charging demand. We further denote T^{base} to be the difference between the EV arrival time and its EDT.

Definition 2: Contract Extend Period T^{extend}: the time difference between EDT and the contract end time.

Definition 3: Delay Tolerance T^{delay}: the time difference between EDT and the actual departure time.

A. Charging Price Menu

We first revisit the conventional charging price scheme widely adopted by existing EVCSs. The conventional charging price consists of two parts, the fixed charging fee (including electricity price and service surcharges) and the overstay fee. The first part is proportional to the EV charging demand, and the second part is proportional to the EV's delay.

Specifically, for an EV with charging demand D and the delay tolerance T^{delay}, the overall charging cost

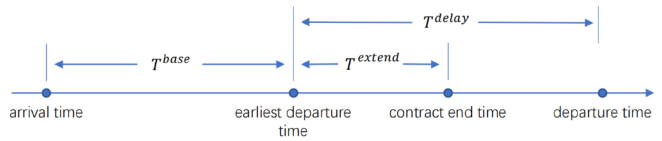


Fig. 2. Illustration of the service process.

$C_{conv}(D, T^{delay})$ with conventional pricing is as follows:

$$C_{conv}(D, T^{delay}) = \xi^{-1} D P_0 + T^{delay} C_{delay}, \quad (1)$$

where ξ is the charging efficiency satisfying $0 < \xi < 1$. P_0 denotes the unit charging fee designed by the EVCS ($\$/KWh$); C_{delay} denotes the unit overstay fee ($\$/h$).

The deadline differentiated dynamic charging price menu, on the other hand, seeks to exploit the flexibility during EV charging. For an EV with charging demand D and maximal charging rate $p^{\max}$, it will take at least T^{base} time slots to meet the charging demand:

$$T^{base} = \left\lceil (\xi p^{\max})^{-1} D \right\rceil. \quad (2)$$

The price menu provides more options for EVs to choose the time completing charging service in exchange for a lower price. If the EV provides more flexibility during charging, it will receive a cheaper charging fee. Specifically, the price menu $\mathcal{M}$ contains $H+1$ time-price pairs[1] as follows:

$$\mathcal{M} = \{(0, P_0), (1, P_1), \ldots, (H, P_H)\}. \quad (3)$$

Each pair (k, P_k) in $\mathcal{M}$ denotes a service contract between the EVCS and the EV: The EVCS commits to satisfy the charging demand D within $T^{base} + k$ time slots; and the EV should stay at the EVCS for at least $T^{base} + k$ slots to enjoy the unit charging fee P_k.

The aforementioned definitions are illustrated in Figure 2. Clearly, the contract extend period T^{extend} is exactly the k that the EV chooses in the price menu. Note that the unit charging fee for $k = 0$ should equal the conventional unit charging price, as the conventional scheme is a special case of the menu-based scheme with $H = 0$, and the conventional charging price scheme is also a part of the menu-based price scheme (option $(0, P_0)$). The designed charging price menu should also satisfy a partial order relationship that, the unit charging fee with more flexibility to complete charging should be cheaper. Formally: $P_i < P_j$ for all $i > j$, where i and j correspond to the indices of the options in the price menu, e.g., (i, P_i).

On the other hand, the dynamic price menu does not influence the overstay fee. That is, for EVs with different menu choices and the same delay tolerance T^{delay}, they should pay the same overstay fees. Therefore, for an EV with demand D, delay tolerance T^{delay}, and menu choice (k, P_k), its overall cost $C_{menu}(D, T^{delay}, k)$ can be calculated as follows:

$$C_{menu}(D, T^{delay}, k) = \xi^{-1} D P_k + T^{delay} C_{delay}. \quad (4)$$

With such a design, we can observe that the EVs actually have more options under the price menu. Based on the price menu, the utility of any EV is no smaller than that of the conventional price scheme. Therefore, all the EVs have incentives to accept such a price menu. Further, due to the existence of the overstay fees, the EVs will not stay at the EVCS for too long.

[1]The parameter H can be adjusted based on the EV's general acceptance of menu length.

B. EV Choice Model

Suppose there are N EVs in the EVCS. Each EV i arrives at the EVCS at time slot t_i with charging demand D_i, which can be automatically obtained when the EV plugs in the charging port. Except for these observable parameters, EVs have different utilities for different departure times. Denote the temporal utility for EV i by $\mathcal{U}_i$. Specifically, $\mathcal{U}_i = [U_{i,0}, U_{i,1}, U_{i,2}, \ldots, U_{i,H}]$ corresponds to $T^{delay} = 0, 1, \ldots, H$. Hence, the overall utility Q_i for EV i with delay tolerance T_i^{delay} when choosing service k is as follows:

$$Q_i(D_i, T_i^{delay}, k) = U_{i,k} - C_{menu}(D_i, T_i^{delay}, k). \quad (5)$$

We make the following assumptions about the EVs:

- EVs have full knowledge of their own utility function.
- EVs are rational and will optimally choose the price options to maximize their utilities.

Therefore, each EV i seeks to maximize its utility by selecting the optimal k_i^* as follows:

$$k_i^* = \arg\max_k \quad Q_i(D_i, T_i^{delay}, k). \quad (6)$$

Such a deadline differentiated price menu can incentivize EVs to truthfully reveal their departure time:

Theorem 1: For any deadline differentiated price menu satisfying partial order relationship, each EV i will truthfully reveal its departure time which equals the contract end time it chooses, i.e., $T_i^{delay} = k_i^*, \forall i$.

This theorem can be proved by the following observation: On the one hand, any EV should not leave before the contract end time due to the requirement of the menu contract. On the other hand, if any EV were to depart later than the contract end time, it could have chosen the contract with a later contract end time for a lower charging price. This observation implies that, EVs are incentivized to truthfully reveal their departure times. This makes it possible for EVCSs to know the EV's departure time in advance, enabling us to further design the optimal price menu.

C. EVCS Model

The EVCS provides charging services to the EVs for maximal revenue. EVCS's revenue is the difference between the total payments from the EVs and the electricity purchasing cost, assuming that the EVCS participates in a real-time electricity market and purchases the electricity at the real-time price. The unit electricity price at time t is denoted by c_t.

During the process of serving EV i with menu choice k_i^*, the revenue R_i for EVCS is as follows:

$$R_i(k_i^*) = \xi^{-1} P_{k_i^*} D_i - E(t_i, k_i^*, \xi^{-1} D_i), \quad (7)$$

where $\xi^{-1} P_{k_i^*} D_i$ represents the service income, and $E(t_i, k_i^*, \xi^{-1} D_i)$ denotes the energy purchase cost. Note that, the overstay fee is used to cover the opportunity cost for extending the time serving EVs.[2] Therefore, it is not included in the revenue. To minimize the energy purchase cost, the following condition holds:

$$\begin{aligned} E(t_i, k_i^*, \xi^{-1}D_i) = \min_{p_i} & \sum_{u=1}^{T_i^{base}+k_i^*} c_{t_i+u} p_{t_i+u} \\ s.t.\ & 0 \le p_i \le p^{\max}, \\ & \sum_{u=1}^{T_i^{base}+k_i^*} p_{t_i+u} = \xi^{-1}D_i. \end{aligned} \tag{8}$$

Note that $E(t_i, k_i^*, \xi^{-1}D_i)$ is associated with the linear programming. Hence, it satisfies the following properties [18]:

Theorem 2: $E(t, k, \xi^{-1}D)$ is piece-wise linear, increasing, and convex in $\xi^{-1}D$. It decreases in k.

This theorem reveals important structure of function $E(t, k, \xi^{-1}D)$ in $\xi^{-1}D$, which enables the fast construction of the function. $E(t, k, \xi^{-1}D)$'s decreasing in k indicates that R_i increases in k. This is intuitive as more flexibility to purchase energy at a lower real-time price can reduce the total purchase cost.

Remark 1: To simplify the price menu design, we assume that all the charging ports in the EVCS share the same maximal charging rates. For charging ports with different charging rates, we can deal with them separately. We also assume the real-time price can be accurately estimated by advanced machine learning technology. We will demonstrate in the experiment that even with inaccurate prediction, the impact of prediction error on the price revenue of EVCS is limited.

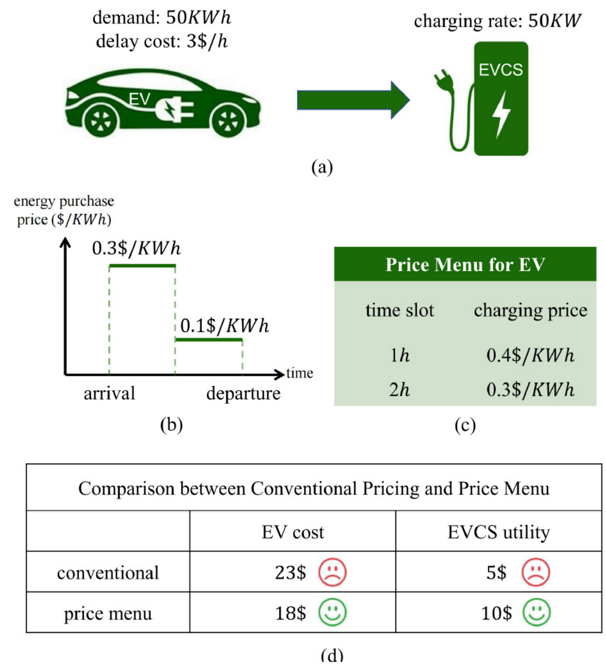
D. Illustrating Example

Fig. 3 provides an illustrating example to highlight the efficiency of the price menu. Consider that an EV of charging demand $50KWh$ wants to leave in 2 hours, and the maximal charging rate in the EVCS is $50KW$, as shown in Fig. 3 (a). Hence, the charging demand can be satisfied in 1 hour. The real-time electricity price is $\$0.3/KWh$ in the first hour and $\$0.1/KWh$ in the second hour illustrated in Fig. 3 (b).

Assume that the conventional charging price offered by the EVCS is $\$0.4/KWh$ and the overstay price is $\$3/h$. The EV should pay at least $0.4*50+3*(2-1) = \$23$, and the EVCS earns $(0.4 - 0.3) * 50 = \$5$ by serving this EV. However, if the EVCS offers an alternative choice (the price menu in Fig. 3 (c)) to satisfy the demand within 2 hours with a charging price of $\$0.3/KWh$, the EV will choose this option with a total payment of $0.3*50+3*(2-1) = \$18$. The EVCS can charge the EV in the second hour at a lower electricity price $\$0.1/KWh$, and its revenue becomes $(0.3 - 0.1) * 50 = \$10$, which is illustrated in Fig. 3 (d). We can observe that the price menu jointly improves the utility of EV and the revenue of the EVCS.

In the subsequent sections, we will introduce how to design the optimal price menu with complete and incomplete information to maximize the EVCS's revenue.

[2]We offer detailed justifications for this argument in Appendix D.



Price Menu for EV	
time slot	charging price
1h	0.4$/KWh
2h	0.3$/KWh

Comparison between Conventional Pricing and Price Menu		
	EV cost	EVCS utility
conventional	23$ ☹	5$ ☹
price menu	18$ ☺	10$ ☺

Fig. 3. Comparison of conventional price and price menu.

III. Price Menu Design With Complete Information

In this section, we first consider the case that the EVCS has complete information on the EVs' utility functions. We propose a constructive price menu design method and investigate the properties of the designed optimal price menu.

A. ϵ-Greedy Optimal Price Menu Design

With complete utility information about the EVs, the EVCS can customize the price menu for each EV. Since the design of the price menu for different EVs does not affect each other, we can focus on the price menu design for a single EV.

For EV i with utility $\mathcal{U}_i = [U_{i,0}, U_{i,1}, U_{i,2}, \ldots, U_{i,H}]$, we can design an ϵ-greedy optimal price menu via the following process. First, we set price $P_{i,k}$ with contract extend period k satisfying the following conditions:

$$U_{i,k} - \xi^{-1}P_{i,k}D_i - kC_{delay} = U_{i,0} - \xi^{-1}P_0D_i, \forall k. \tag{9}$$

Mathematical manipulation yields that:

$$P_{i,k} = P_0 - \frac{\xi(U_{i,0} - U_{i,k} + kC_{delay})}{D_i}, \forall k. \tag{10}$$

This design guarantees that EV i will get the same utility with all menu choices. To make EV i strictly prefer option k, the EVCS only needs to slightly decrease $P_{i,k}$ as follows:

$$P_{i,k} = P_0 - \frac{\xi(U_{i,0} - U_{i,k} + kC_{delay})}{D_i} - \epsilon, \tag{11}$$

where $\epsilon > 0$ is small. The EVCS's utility $R_i(k)$ is as follows:

$$R_i(k) = P_{i,k}D_i - E(t_i, k, D_i), \forall k \tag{12}$$

To maximize its utility, the optimal choice k_i^* satisfies:

$$k_i^* = \arg\max_k \quad R_i(k). \tag{13}$$

Therefore, the ϵ-greedy optimal price menu $\mathcal{M}_i = \{(k, P_{i,k}), \forall k\}$ is as follows:

$$P_{i,k} = \begin{cases} P_0 - \frac{\xi(U_{i,0}-U_{i,k}+kC_{delay})}{D_i}, & k \neq k_i^*, \\ P_0 - \frac{\xi(U_{i,0}-U_{i,k}+kC_{delay})}{D_i} - \epsilon, & k = k_i^*. \end{cases} \quad (14)$$

B. Property Analysis

Define any price menu which maximizes EVCS's revenue as the optimal price menu. We first prove the proposed ϵ-greedy price menu is an optimal price menu. Then, we analyze the common properties of optimal price menus.

Theorem 3: The ϵ-greedy price menu with complete information can maximize the EVCS's profit when $\epsilon \to 0$.

This theorem indicates that with complete information, we can design an optimal price menu through simple construction rather than solving complex optimization problems.

Before analyzing the impact of optimal price menus on the EVs, we first introduce the following notion:

Definition (First-Degree Price Discrimination) [19]: The first-degree price discrimination involves the seller charging a different price for each unit of the good to different customers in such a way that the price charged for each unit equals the maximum willingness to pay for that unit.

The first-degree price discrimination emphasizes that the seller has complete information about the customers' willingness to purchase. Based on such complete information, the seller can always maximize its own utility [19].

For the EV charging, we can show that the optimal price menus are, in fact, first-degree price discrimination. Formally:

Theorem 4: Any optimal price menu in terms of maximizing social welfare causes first-degree price discrimination.

This theorem indicates that, although an optimal price menu can maximize both EVCS's own revenue and the social welfare, the utility improvement of the EV is completely ignored. All the increased social welfare becomes the EVCS's revenue. As a result, the EV's consumer surplus is zero, and first-degree price discrimination happens. This discourages EVs from joining the menu-based price scheme.

IV. Price Menu Design With Incomplete Information: The Basis

In practice, we can hardly observe the utility information of each EV, rendering the customized price menu for each EV impossible. However, it is possible to obtain the distribution of utility functions across the EV population, enabling the design of a uniform dynamic price. That is, the EVCS offers the same price menu for all the EVs arriving at the same time. This is also more practical and fair. In this section, we first design an approach to estimate the distribution of EV population's utility and demand based on EV behavior patterns. Further, we use the clustering algorithm to efficiently construct the price menu.

A. EV Information Estimation

We use EV population $\mathcal{P}$'s behavior data [3] to estimate their utility functions. First, we assume EV i's utility function $U_i(t)$ is concave, where t denotes the time since the minimal charging end time.

One commonly used form is the quadratic function [15], [20], e.g., $U_i(t) = \alpha_i t(\beta_i - t)$. In this form, the factor α_i can capture the EV's urgency to charge and its unit time value, while β_i can reflect temporal preference. We use this quadratic form to better illustrate our subsequent information estimation. However, we want to emphasize that our analysis can be extended to general concave utility function $U_i(t)$.

For each EV i, we seek to estimate the corresponding tuple $\mathcal{G}_i = \{t_i, \alpha_i, \beta_i, D_i\}$. The arrival time t_i and charging demand D_i can be directly obtained from charging session data. To estimate α_i, which is related to EV's unit time value, we jointly utilize EV's static time value and dynamic time value. The static time value reflects EV's average time value and is linear to EV driver's income. Intuitively, this assumption indicates that higher income yields less sensitivity to the charging cost. When estimating the dynamic time value, we assume that an EV driver with a more complex daily trajectory has a higher dynamic time value. We use the spatial-temporal trajectory entropy [21] to evaluate the complexity of the trajectory. Thus, α_i can be estimated as follows:

$$\alpha_i = I_i + ST_i, \quad (15)$$

where I_i is the average time value, and ST_i is the spatial-temporal trajectory entropy which is the weighted sum of spatial trajectory entropy ST_i^S and temporal trajectory entropy ST_i^T:

$$ST_i^S = -\sum_{j=1}^{W_i}\left(D_{i,j} \cdot \frac{T_{i,j}}{\sum_{j=1}^{W_i} T_{i,j}} log_2\left(\frac{T_{i,j}}{\sum_{j=1}^{W_i} T_{i,j}}\right)\right), \quad (16)$$

$$ST_i^T = -\sum_{j=1}^{W_i}\left(T_{i,j} \cdot \frac{T_{i,j}}{\sum_{j=1}^{W_i} T_{i,j}} log_2\left(\frac{T_{i,j}}{\sum_{j=1}^{W_i} T_{i,j}}\right)\right). \quad (17)$$

Note that W_i is the number of visited places of EV i. $D_{i,j}$ represents the travel distance of EV i to orderly visit the next place from j^{th} place. $T_{i,j}$ represents the duration of stay in j^{th} visited place of EV i.

Note that, β_i is the utility maximizer for EV i. Hence, $\frac{\beta_i}{2}$ is EV's most desired time to depart. We use the EV's actual departure time from the EVCS to estimate $\frac{\beta_i}{2}$.

B. Information Clustering and Calculation

The information tuples for all EVs leave us with the data of huge volume, which will significantly burden the dynamic

[3] The behavior data and the subsequent price menu design mainly target to a small region, i.e., Berkeley downtown in the great San Francisco area. Hence, we assume the EV charging demands and EV user patterns are similar to different EVCSs in a small region. And the large volume of data on a regional scale enable more accurate charging pattern estimation. Further, our framework enables the customized price menu design for a single EVCS by collecting the charging session data and EV information of the corresponding EVCS.

price menu design in real-time. To tackle this problem, we separate the information tuples according to the arrival time. For each tuple group with the same arrival time t, we can set the number of target clusters to be N_c^t. Then we use the K-means clustering algorithm to cluster the information data into N_c^t groups. For each group, we use the mean of the information data in the group to represent its patterns. We can also obtain the proportion of each pattern. Specifically, through the clustering process, we construct the information set $\hat{\mathcal{S}}_t$ for different t as follows:

$$\begin{aligned}\hat{\mathcal{S}}_t = ((&\hat{\alpha}_{t,1}, \hat{\beta}_{t,1}, \hat{D}_{t,1}, \hat{\sigma}_{t,1}), \ldots,\\ &(\hat{\alpha}_{t,N_c^t}, \hat{\beta}_{t,N_c^t}, \hat{D}_{t,N_c^t}, \hat{\sigma}_{t,N_c^t})),\end{aligned} \tag{18}$$

where $\hat{\alpha}_{t,i}$, $\hat{\beta}_{t,i}$, $\hat{D}_{t,i}$ and $\hat{\sigma}_{t,i}$ denote the estimation for the i^{th} EV pattern at time t after clustering.

Thus, we can calculate the utility $\hat{\mathcal{U}}_i^t$ for each EV pattern i arriving at time slot t according to $\hat{\mathcal{S}}_t$. Specifically, $\hat{\mathcal{U}}_i^t = [\hat{U}_{i,0}^t, \ldots, \hat{U}_{i,H}^t]$.

C. Extended EV Information Estimation

The estimation method mentioned above doesn't differentiate the EV user patterns on different days in a week. However, in practice, the aggregate EV charging demand profile may vary greatly between different days, and so do the EV information patterns. Without differentiating different days in a week, it would be difficult to capture the temporal characteristics of EV information patterns.

To address this issue, we extend the existing method and modify the EV pattern estimation to obtain the EV user information over different days, which further helps design the more customized price menus. Specifically, we first propose to design a more realistic day-differentiated price menu (i.e., design different price menus for Monday, Tuesday, etc.), and then extend it to a more general demand profile-differentiated price menu design to capture the varying demands.

1) Day-Differentiated EV Information Estimation: To facilitate the design, we only need to slightly change the original estimation process. The existing method separates the information tuples based on the arrival time, whereas the day-differentiated price menu design requires separating the information tuples based on both the day type and the arrival time. For each tuple group on the same day-type w with arrival time t, we can set the number of target clusters to be $N_c^{w,t}$. Then we use the K-means clustering algorithm to classify the information data into $N_c^{w,t}$ groups. This gives us the corresponding pattern for each group. Consequently, we can obtain 7 groups of EV information patterns, and each group contains the EV information with the same scale as the original results in Eq. (18).

2) Demand Profile-Differentiated EV Information Estimation: The drawback of the day-differentiated price is that, the EV charging demand may also vary dramatically on the same day of different weeks. This further inspires us to design a demand profile-differentiated EV information estimation. Specifically, we first conduct the K-means clustering algorithm to classify different days according to the daily demand profile. Then we assign different day types w to the EV information tuples in different clusters. The demand-profile differentiated patterns can be obtained following the same routine as mentioned above.

V. Price Menu Design With Incomplete Information: Implementation

In this section, we seek to solve the deadline differentiated dynamic charging price menu based on the estimated EV utility information. The price menu design across different time slots can be decoupled. Hence, we only concentrate on the design in a single time slot.

A. Bi-Level Optimization Formulation

The price menu design problem can be formulated into a bi-level optimization. In the upper level, the EVCS designs the price menu to maximize its revenue, while in the lower level, EVs decide their menu choices according to the price menu.

1) Upper Level: Denote the price menu to be designed at time t as $\mathcal{M}_t = \{(0, P_{t,0}), (1, P_{t,1}), \ldots, (H, P_{t,H})\}$. The upper level optimization (**UL**) is as follows:

$$\textbf{(UL)} \max_{P_{t,k}, k_{t,i}^*} \sum_{i=1}^{N_c} \sigma_{t,i} \left(\xi^{-1} P_{t,k_{t,i}^*} \hat{D}_{t,i} - \sum_{u=1}^{T_{t,i}^{base}+k_{t,i}^*} c_{t+u} p_{i,t+u} \right)$$

$$s.t. \quad 0 \le p_{i,t+u} \le p^{\max}, \forall i, u, \tag{19}$$

$$\sum_{u=1}^{T_{t,i}^{base}+k_{t,i}^*} p_{i,t+u} = \xi^{-1} \hat{D}_{t,i}, \forall i, \tag{20}$$

$$P_{t,0} \le P_0, \tag{21}$$

$$P_{t,k+1} \le P_{t,k}, \forall k, \tag{22}$$

where $P_{t,k}$, $k_{t,i}^*$, and $p_{i,t+u}$ are the decision variables. $P_{t,k}$ denotes the unit charging price with contract extend period k for the EVs arriving at time t; $k_{t,i}^*$ denotes the menu choice of i^{th} EV pattern arriving at time t, which is an integer random variable, and $p_{i,t+u}$ denotes the charging rate of i^{th} EV pattern at time $t+u$.

The objective function denotes the total revenue of the EVCS for serving the EVs arriving at time t. Constraint (19) denotes the charging rate limit. Constraint (20) enforces satisfying the charging demand of each EV pattern. Constraint (21) indicates that the menu-based price should be lower than the conventional price. Constraint (22) ensures that the menu price is decreasing in the contract extend period.

2) Lower Level: The lower level captures the coupling between the price menu $P_{t,k}$ and EVs choice $k_{t,i}^*$.

$$\textbf{(LL)}\ k_{t,i}^* = \arg\max_k \quad \hat{U}_{i,k}^t - \xi^{-1} \hat{D}_{t,i} P_{t,k} - k C_{delay}, \forall i. \tag{23}$$

Problem **LL** includes integer variable $k_{t,i}^*$, and the bi-level structure makes the whole optimization even more intractable.

B. Mixed-Integer Quadratic Programming Formulation

Next, we transform the intractable bi-level optimization into mixed-integer quadratic programming, which can be solved efficiently with commercial solvers, e.g., Gurobi [22].

1) Constraints Elimination for UL Optimization: We can observe two facts from **UL**: First, the constraints (19) and (20) do not include the decision variables $P_{t,k}$ and $k^*_{t,i}$, and are only related to $p_{i,t+u}$. Further, $p_{i,t+u}$ does not couple with $P_{t,k}$ in the objective function. Therefore, the **UL** optimization can be transformed into the form free of $p_{i,t+u}$:

$$(\textbf{RUL}) \quad \max_{P_{t,k},k^*_{t,i}} \sum_{i=1}^{N_c} \sigma_{t,i}\Big(\xi^{-1}P_{t,k^*_{t,i}}\hat{D}_{t,i} - \hat{B}(t,i,k^*_{t,i})\Big)$$
$$s.t. \quad \textbf{Constraints} \quad (21)-(22). \tag{24}$$

where $\hat{B}(t,i,k)$ is the following function:

$$\hat{B}(t,i,k) = \min_{p_{i,t+u}} \sum_{u=1}^{T^{base}_{t,i}+k^*_{t,i}} c_{t+u}p_{i,t+u}$$
$$s.t. \quad 0 \le p_{i,t+u} \le p^{\max}, \forall u, \tag{25}$$
$$\sum_{u=1}^{T^{base}_{t,i}+k^*_{t,i}} p_{i,t+u} = \xi^{-1}\hat{D}_{t,i}.$$

Note that, problem (25) can be solved in advance for all possible t, i, and k.

*2) Reformulation of **LL** Optimization:* The most tricky hurdle in the **UL** optimization is that, k^*_i is included in the upper limit of the summation. To mitigate this issue, we reformulate the **LL** problem into the following optimization with binary variables:

$$(\textbf{RLL}) \quad \min_{a_{t,i,k}} \sum_{k=1}^{H} a_{t,i,k}(\hat{U}^t_{i,k} - \xi^{-1}\hat{D}_{t,i}P_{t,k} - kC_{delay})$$
$$s.t. \quad \sum_{k=1}^{H} a_{t,i,k} = 1, \forall i, \tag{26}$$
$$a_{t,i,k} \in \{0,1\}, \forall i,k. \tag{27}$$

Note that $a_{t,i,k}$ is the binary indicator for whether the EV pattern i arriving at time t chooses k^{th} service: $a_{t,i,k} = 1$ only when $k^*_{t,i} = k$.

3) Combined Optimization: Problem **RUL** and **RLL** can be efficiently combined through the classical Big M approach as follows:

$$(\textbf{CO}) \quad \max_{P_{t,k},a_{t,i,k}} \sum_{i=1}^{N_c}\sum_{k=1}^{H} \sigma_{t,i}a_{t,i,k}(P_{t,k}\xi^{-1}\hat{D}_{t,i} - \hat{B}(t,i,k))$$
$$s.t. \quad \textbf{Constraints} \quad (21)-(22),$$
$$\textbf{Constraints} \quad (26)-(27),$$
$$\hat{U}^t_{i,k} - \frac{1}{\xi}P_{t,k}\hat{D}_{t,i} - kC_{delay} + \sum_{q\neq k} a_{t,i,q}M$$
$$\ge \hat{U}^t_{i,j} - \frac{1}{\xi}P_{t,j}\hat{D}_{t,i} - jC_{delay}, \forall i, k \neq j, \tag{28}$$

where M is a very large positive number. Constraint (28) guarantees EVs' rationality to maximize their own revenue.

C. Revenue Transfer Fee Design

The proposed framework in the former sections has two major drawbacks. It cannot guarantee the same revenue for EVCS as that in the complete information case due to the missing EV information during clustering. Note that, even without clustering, the revenue of EVCS may fluctuate a lot.

In this section, we propose a uniform revenue transfer fee [23] based on the population information (instead of the clustered information) to offset the social welfare of different groups. Specifically, the EVCS can design a social welfare allocation target ratio θ_t ($0 \le \theta_t \le 1$) where only θ_t portion of the increased social welfare will be dispatched to the EVCS. Denote the desired revenue transfer fee at time t by δ_t. The following conditions relate θ_t and δ_t:

$$U^{EVCS}_{t,conv} + U^{EV}_{t,conv} + SW_t = U^{EVCS}_{t,menu} + U^{EV}_{t,menu}, \tag{29}$$
$$U^{EVCS}_{t,conv} + \theta_t SW_t = U^{EVCS}_{t,menu} + \sum_{i\in\mathcal{P}} \xi^{-1}D_{t,i}\delta_t, \tag{30}$$

where $U^{EVCS}_{t,conv}$, $U^{EVCS}_{t,menu}$ denote the revenue for EVCS by serving the EV population under conventional and menu-based pricing scheme at time t; $U^{EV}_{t,conv}$, $U^{EV}_{t,menu}$ denote the revenue for EV population under conventional and menu-based pricing scheme at time t; and SW_t denotes the increased social welfare introduced by EVs' flexibility. Mathematical manipulation yields that:

$$\delta_t = \frac{(1-\theta_t)(U^{EVCS}_{t,conv} - U^{EVCS}_{t,menu}) + \theta_t(U^{EV}_{t,menu} - U^{EV}_{t,conv})}{\sum_{i\in\mathcal{P}} \xi^{-1}D_{t,i}}, \tag{31}$$

where each revenue term can be calculated directly as follows:

$$U^{EVCS}_{t,conv} = \sum_{i\in\mathcal{P}} (\xi^{-1}D_{t,i}P_0 - B(t,i,0)), \tag{32}$$
$$U^{EVCS}_{t,menu} = \sum_{i\in\mathcal{P}} (\xi^{-1}D_{t,i}P_{k^*_{t,i}} - B(t,i,k^*_{t,i})), \tag{33}$$
$$U^{EV}_{t,conv} = \sum_{i\in\mathcal{P}} (U^t_{i,0} - \xi^{-1}D_{t,i}P_0), \tag{34}$$
$$U^{EV}_{t,menu} = \sum_{i\in\mathcal{P}} (U^t_{i,k^*_{t,i}} - k^*_{t,i}C_{delay} - \xi^{-1}D_{t,i}P_{k^*_{t,i}}), \tag{35}$$

where $D_{t,i}$, $B(t,i,k^*_{t,i})$, $U_{i,k^*_{t,i}}$ denote the corresponding demand, electricity purchasing cost and EV utility for the EV i at time t in EV population (not clustered patterns). The final derived price menu satisfies that $\mathcal{M}_t = \{(k, P_k + \delta_t), \forall k\}$. With the revenue transfer fee, we detail the dynamic deadline differentiated price menu design in Algorithm 1.

D. Extension to More Customized Price Menu

In the above sections, we have introduced how to design a time-varying dynamic price menu. With this price menu, all EVs arriving at the same time receive exactly the same price menus. However, we know that EV's charging demand is another key parameter to influence the charging process and the price menu design. Particularly, it is an observable parameter in contrast to EV utility. Next, we seek to design customized price menus for EVs arriving at the same time but with different charging demands. That is, the extended price menu is a function of both arrival time and charging demand.

In fact, we can obtain the extended price menu by making a few minor adjustments to the aforementioned price menu design framework. Intuitively, the aforementioned framework

Algorithm 1 Dynamic Price Menu Design

Input Clustered EV information $\hat{\mathcal{S}}_t$ at each time t; The price menu length H; The desired social welfare allocation target ratio θ_t at each time t; The price prediction window size W;
Output Price menu $\mathcal{M}_t = \{P_{t,0}, ..., P_{t,H}\}$ for each t;
for $t = 0, 1, ...$ **do**
  Predict the future price $\mathcal{C}_t = \{c_{t+1}, c_{t+2}, ..., c_{t+W}\}$;
  for $i = 1, 2, ..., N_c^t$ **do**
    Calculate driver pattern i's utility $\hat{\mathcal{U}}_i^t$ according to EV information $\hat{\mathcal{S}}_t$;
    for $k = 1, 2, ..., H$ **do**
      Calculate $\hat{B}(t, i, k)$ via optimization (25)
    end for
  end for
  Solve the optimal menu $\mathcal{M}_t$ according to (**CO**);
  Calculate the revenue transfer fee δ_t;
  return $\mathcal{M}_t = \{(k, P_k + \delta_t), \forall k\}$;
end for

first groups EVs according to the arrival time, and then designs the price menu customized for each EV group. To extend to price menus for EVs with different charging demands, we can simply divide population $\mathcal{P}$'s information data into different groups according to both arrival time and charging demand. Then we can straightforwardly follow our proposed price menu design process, i.e., clustering and then optimization, to calculate the optimal price menu for each group.

VI. Numerical Study

In this section, we first estimate the utility and demand distribution of EV population, and then evaluate the performance of our designed price menu in terms of revenue, computational efficiency, grid impact and robustness. The numerical study is conducted on a laptop with Intel i5-8265U CPU @ 1.60 GHz and 8GB RAM, and we adopt the Gurobi solver 9.5.0 [22].

A. Data Description

We estimate EVs' charging behaviors based on four independent data sources: mobile phone activity data of Bay area residents [24], charging sessions obtained from the commercial EV supply equipment in the same region [25], surveys on the use of conventional and electric vehicles [26], together with census data for income information at 5-digit Zipcode resolution [27]. We adopt methodology in [28] and [29] to obtain the spatial-temporal trajectory, energy demand at arrival, the start charging time, the end charging time, the departure time, and the personal income of 70, 736 Bay area EVs. The spatial distribution of EVs are illustrated in Fig. 4 (a). We use 15 minute average electricity price data in 2021 offered by California ISO illustrated in Fig. 4 (b).

Besides, we select the length of each time slot to be 15 minutes. The charging price of the EVCS is upper bounded by $\$0.05/KWh$ (do not include service surcharges), and the overstay price is set to be $\$2/h$. The maximal charging rate is $13.2KW$. The charging efficiency is set to be 0.95.

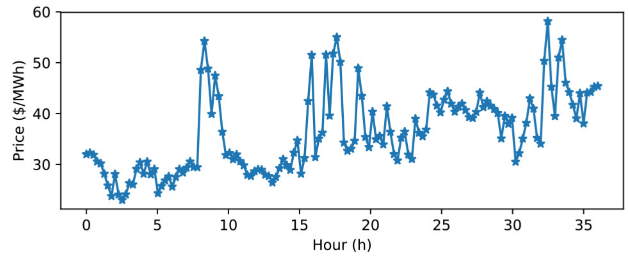


Fig. 4. Electricity Price.

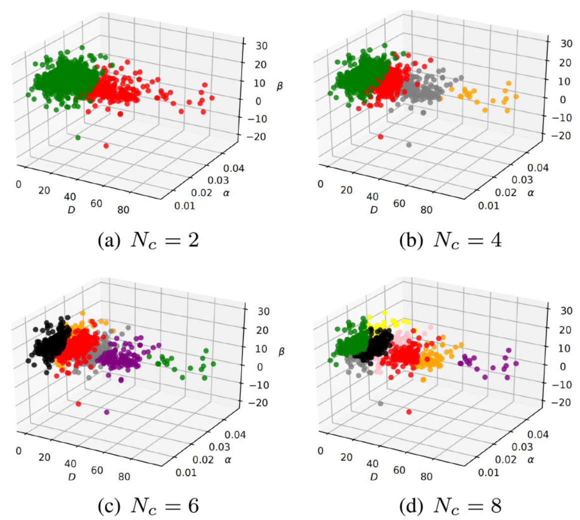


Fig. 5. Information Clustering.

B. EV Information Estimation

We repeat the K-means clustering algorithm with different initial points for 10 times, which leads to similar clustering results, demonstrating the stability of our method. Fig. 5 illustrates the clustering result with N_c to be $2, 4, 6, 8$. We can observe that when N_c is small, the dominant dimension is EV charging demand. And when N_c increases, α and β gradually affect the clustering results, and the clustered EV patterns become more diverse. By trading off the impacts of clustering performance and algorithm running time, we select N_c to be 8 in the subsequent experiments.

For more intuitions, we visualize the pattern information with $N_c = 8$ in Fig. 6. The color of circles in each subfigure represents the charging demand of EV patterns according to the color bar. The blue lines represent EV patterns' utility functions considering overstay fees with respect to the contract extend period. We find that the utilities of most EVs with small charging demands in Fig. 6 (a), (d), (e) first increase and then decrease, indicating that they desire a later departure time, which corresponds to more flexibility to the EVCS. While for EVs with large charging demands in Fig. 6 (f), (g), (h), their utility functions drop rapidly, which indicates these EVs want to leave as soon as possible after a long charging session, corresponding to less flexibility.

C. Price-Menu Revenue Evaluation

Next, we evaluate the performance of our proposed deadline differentiated price menu. We simulate the EV charging

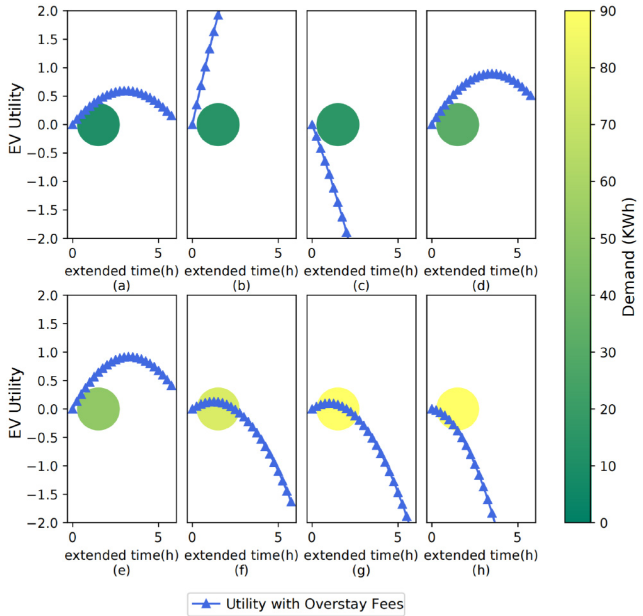


Fig. 6. Typical Patterns.

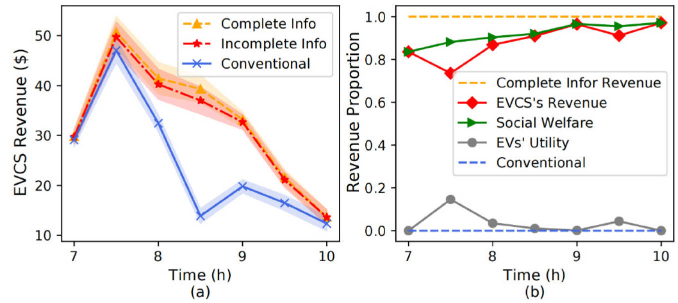


Fig. 7. Utility and Social Welfare.

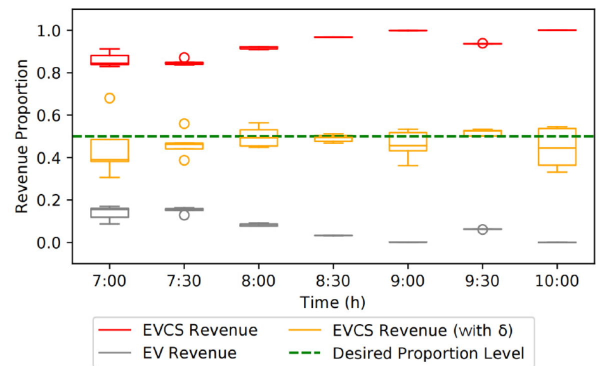


Fig. 8. Power of Revenue Transfer Fee.

process during the morning peak period (7:00 am to 10:00 am) for 10 times, and EVs are randomly sampled from the population according to their arrival time. Fig. 7 illustrates its performance compared with conventional pricing and menu-based pricing with complete information. In Fig. 7 (a), we can observe that our price menu under incomplete information can achieve nearly the same revenue with complete information. Fig. 7 (b) visualizes the proportions of social welfare of EVCS and EVs benchmarking with the case under complete information. We can observe that the total social welfare with incomplete information (the green line) is always above 80%. However, although both EVCS and EVs' utilities are improved compared with conventional pricing case (the line), EVCS takes the major part of the social welfare (the red line), and the EVs only obtain a small portion (the grey line), highlighting the need for the revenue transfer fee.

With the introduction of the revenue transfer fee, we set the desired EVCS revenue percentage δ to be 0.5. Fig. 8 compares the proportions of social welfare that EVCS and EVs take before and after introducing the revenue transfer fee. We can find that before introducing the revenue transfer fee, the EVCS dominates about 90% of the social welfare, whereas the EVs only share 10% of the social welfare. With the revenue transfer fee, the EVCS's revenue share is reduced to about 50%, which conforms to the desired proportion level well. Therefore, the EVs can also enjoy about half of the social welfare. It shows that the revenue transfer fee can effectively balance the distribution of social welfare.

Then we evaluate the performance of the more customized price menus for EVs with different charging demands. We group EVs arriving at each time into 2, 3, or 4 categories according to charging demands in population data, ensuring the same number of EVs in each category, and then obtain the price menu and demand range corresponding to each category. We simulate 5 times for each extended price menu during the morning peak. The average revenues of each price menu are provided in Table I. We can observe that the more customized price menu reduces the gap between the time-varying uniform price menu (# groups = 1) and the price menu with complete information (CI) by more than 30%. When the number of categories is larger than 2, increasing the number of categories will not significantly improve the revenue.

TABLE I
REVENUE OF HETEROGENEOUS PRICE MENU

# Group(s)	1	2	3	4	CI
Revenue ($)	402.74	405.96	405.43	405.39	412.64

D. Extended Price Menu Revenue Evaluation

For the day-differentiated price menu design, the weekly charging demand is illustrated in Fig. 9. We can observe that the charging demands are quite different between weekdays and weekends. The weekday charging demand is far larger than that of the weekend since most customers for commercial EVCSs are commuters who charge EVs during working hours.

We visualize the charging revenues of weekdays and weekends from 5:00 am to 9:00 pm in Fig. 10, and the shadowed area stands for the standard deviation. Generally, the revenue curves align with the demand curves, i.e., the weekday revenue is about 5 times to the weekend revenue.

For the demand profile-differentiated price menu design, we conduct the K-means clustering algorithm to classify the days in a year into different groups according to the demand profile, which is illustrated in Fig. 11, and we can observe that the demand profiles in each group are quite similar. Then, we

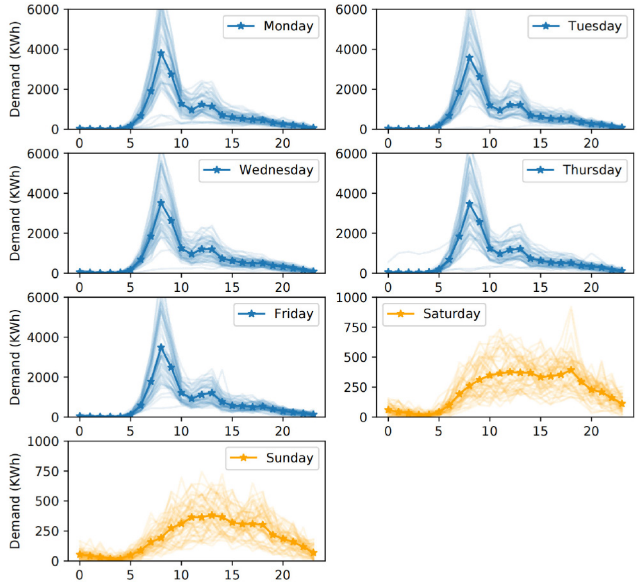


Fig. 9. Weekly Charging Demand.

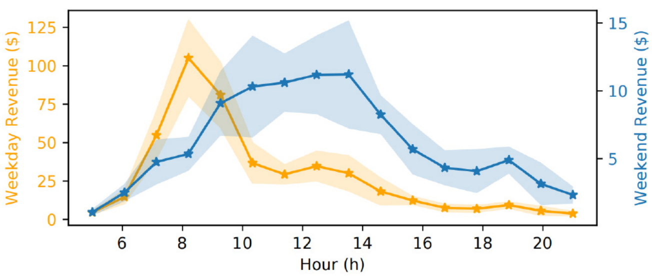


Fig. 10. Weekly Charging Revenue.

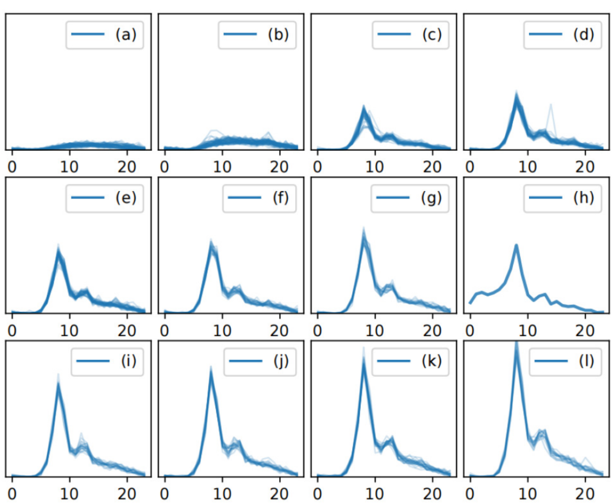


Fig. 11. Demand Pattern Clustering.

follow the routine to calculate the EV information pattern and price menus for each group. Fig. 12 illustrates the customized price menu's performance in terms of the changes in the total revenue with varying number of clusters. We can observe that, the revenue is significantly improved when the number of clusters changes from 1 to 2. When the number of clusters further increases, the revenue improves at a much slower rate.

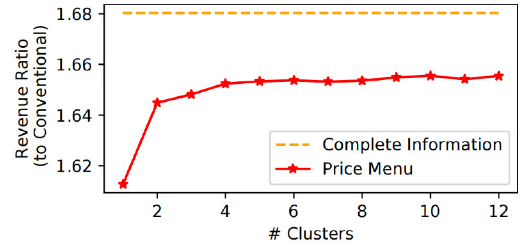


Fig. 12. Performance with Demand Pattern Clustering.

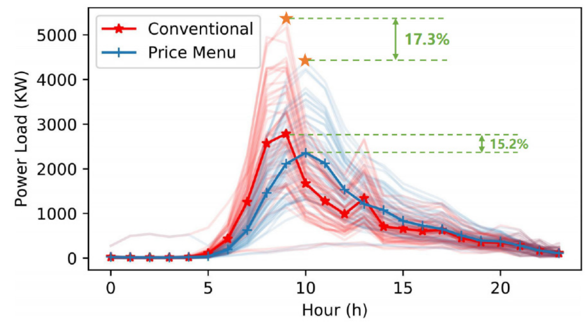


Fig. 13. Charging Load Shifting.

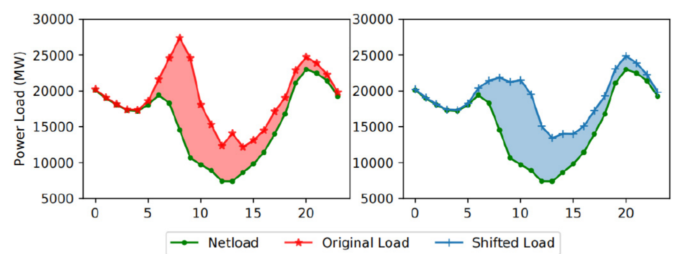


Fig. 14. Peak Load Shaving.

E. Impact on Power Grid

Fig. 13 illustrates the charging load curve with the conventional price scheme and the price menu in 50 different weekdays, [4] and the solid lines characterize the average loads. It is evident that, with the price menu, the morning charging peak from 7:00 am to 9:00 am can be effectively postponed to around 10:00 am. The historical maximum charging load (marked by the yellow stars) in 50 days can be reduced by 17.3%, and the average maximum load is reduced by 15.2%.

Further, to study the impact on the power system, we consider there are 5, 000 EVCSs with similar scales in California. We obtain the baseload data of California on Apr. 26, 2022 from CAISO [30], which is illustrated as the green line in Fig. 14. The red line and blue line show the aggregated loads before and after the price menu. We can observe that the aggregated load of the grid with the conventional price scheme reaches the first peak at 8:00 am. In contrast, with the menu-based price scheme, the gird peak load is effectively reduced, and the load valley (caused by the integration of solar power) is better smoothed. The fluctuations in the grid load during the whole day are also reduced.

[4] We only select weekday load because the weekday load is much higher than that of the weekend.

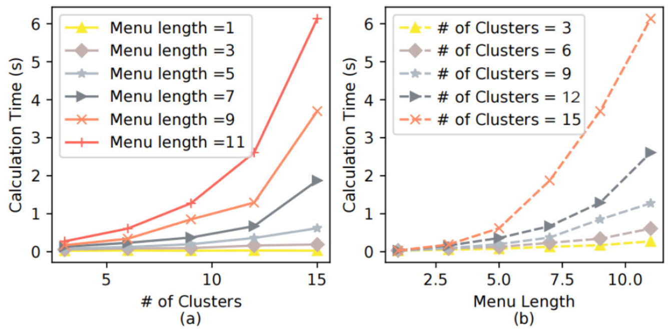


Fig. 15. Computational Efficiency.

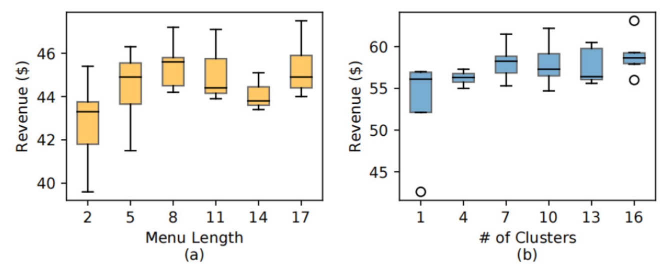


Fig. 16. Parametric Evaluation.

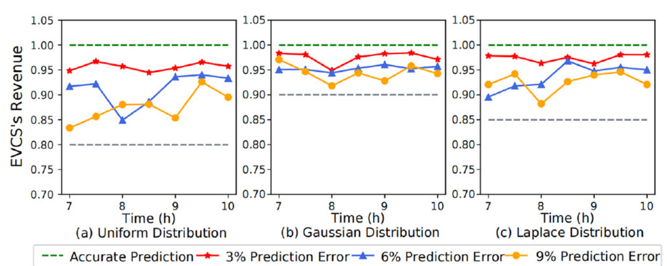


Fig. 17. Performance under Prediction Errors.

F. Computational Efficiency and Robustness Evaluation

To measure computational efficiency, we evaluate the impact of cluster numbers and menu length on running time. We run the experiments with different cluster numbers and menu lengths 10 times each, and plot the mean computation time in Fig. 15. It is obvious that the running time increases with the increase of cluster number and menu length by a superlinear speed due to the complexity of mixed-integer programming. Nonetheless, even with 15 clusters and a menu length of 12 (indicating 3 hours of contract extend period), the price menu can be solved within 6s, demonstrating our approach's efficiency.

Fig. 16 illustrates how the number of clusters and the menu length influence EVCS's utility: the revenue first increases and then is stable as both parameters' increase. The number of clusters is more significant in terms of improving the revenue. As for the menu length, when it is larger than 8, the revenue doesn't grow monotonically but fluctuates instead.

Furthermore, we evaluate the price menu's robustness against electricity price prediction errors. We consider three variants of error distribution, i.e., uniform distribution, Gaussian distribution, and Laplace distribution with mean relative error (MRE) ranging from 3% to 9%. Fig. 17 illustrates the robustness of our price menu. The loss of EVCS's revenue is no larger than 20% even with the worst prediction results.

VII. Conclusion

In this paper, we design the optimal deadline differentiated dynamic price menu with complete and incomplete information. We also propose a systematic method to estimate the utility and demand information for a large population of EVs based on EV behavior to aid the price menu design in incomplete information. The numerical study demonstrates our designed price menu regarding revenue, computational efficiency and robustness.

Our work can be extended in various interesting directions. For example, it is interesting to jointly consider the price menu design and coupled charging control operation among different EVs. It is also interesting to exploit other grouping criteria to achieve a more refined price menu and provide better differentiated services to EVs. Moreover, it is also important to study how to improve the robustness of the price menu for extremely large EV populations.

Appendix

A. Proof for Theorem 3

For any price menu $\mathcal{M}$ with choices (k, P_k). EV's optimal choice k^* should bring it no smaller utility than choice $(0, P_0)$:

$$U_{i,k^*} - \xi^{-1}P_{i,k^*}D_i - k^*C_{delay} \geq U_{i,0} - \xi^{-1}P_0D_i. \quad (36)$$

By comparing with our constructed price menu in (10), we can conclude that when $\epsilon \to 0$, for any menu, no matter what the EV's optimal choice under this menu is, the corresponding price of our constructed menu is always more expensive. It indicates that the revenue of our constructed menu is no less than that of any menu, and thus the optimality holds. This concludes our proof. ■

B. Proof for Theorem 4

By Theorem 2, we know that any optimal price menu will lead to the same EV choice as our constructed price menu. Thus, we only need to focus on the constructed price menu.

With our price menu, the EVCS should decide the optimal k^* to maximize its revenue, i.e.,

$$\begin{aligned} k^* &= \arg\max_k \quad P_0 - \frac{\xi(U_{i,0} - U_{i,k} + kC_{delay})}{D_i} && (37)\\ &= \arg\max_k \quad \xi(U_{i,k} - kC_{delay}) - E(t_i, k, D_i) && (38)\\ &= \arg\max_k \quad Q_{i,k} + R_i(k). && (39) \end{aligned}$$

We can observe that the optimization objective has been transformed into social welfare.

Further, we know that EV's utility is linear in ϵ. And ϵ approaching 0 leads to zero consumer surplus of EV, inducing the first order price discrimination. ■

C. Models of the Opportunity Cost and Overstay Fee

We define the opportunity cost C_{opp} as the marginal charging utility from a single charging port during Δt period. Specifically:

$$C_{opp} = \eta p^{\max} \Delta t(P - c), \tag{40}$$

where η denotes the busy rate of the EVCS, which is defined to be the ratio between charging demand and charging capacity during Δt period. When $\eta \rightarrow 0$, it means that there are very few charging demands, whereas when $\eta \rightarrow 1$, it means that the EVCS is very busy, i.e., nearly all of the charging facilities are occupied. $p^{\max}$ denotes the maximal charging rate of the EV (KW), Δt is the duration of a single time slot (h), P denotes the unit charging fee ($\$/KWh$), and c denotes the unit electricity purchasing price from the grid ($\$/KWh$). Therefore, $p^{\max}\Delta t(P - c)$ stands for the maximal utility of serving a new EV during Δt period. The busy rate η also implies the probability that the charging port can be occupied during Δt period. Hence, multiplying η means the revenue can be earned with probability η. Therefore $C_{opp} = \eta p^{\max}\Delta t(P - c) + (1 - \eta) \times 0 = \eta p^{\max}\Delta t(P - c)$.

To see how the opportunity cost is covered, we can define the unit overstay fee in the corresponding Δt period as follows:

$$C_{delay} = \hat{\eta} p^{\max} \Delta t(P - \hat{c}), \tag{41}$$

where $\hat{\eta}$, $\hat{c}$ denote the EVCS's estimations to the busy rate η and electricity price c, respectively.

During the process of serving EV i with menu choice k_i^*, the unit revenue ΔR_i during Δt period for EVCS is as follows:

$$\begin{aligned}\Delta R_i &= (P_{k_i^*} - c)\xi^{-1}\Delta D_i + C_{delay} - C_{opp} &(42)\\ &= (P_{k_i^*} - c)\xi^{-1}\Delta D_i + p^{\max}\Delta t(\hat{\eta}(P - \hat{c}) - \eta(P - c)), &(43)\end{aligned}$$

where $P_{k_i^*}$ denotes the charging price, ξ^{-1} represents the charging efficiency, ΔD_i denotes the charging amount for EV i during Δt. Therefore, $(P_{k_i^*} - c)\xi^{-1}\Delta D_i$ represents the charging service revenue from the difference between charging price income and electricity purchasing cost, C_{delay} represents the unit overstay fee income, and C_{opp} denotes the unit opportunity cost. We assume the EVCS can accurately predict the future busy rate η and electricity price c based on historical information. Therefore, when $\hat{\eta} = \eta$, $\hat{c} = c$, the designed overstay fee can perfectly cover the opportunity cost, i.e.,

$$p^{\max}\Delta t(\hat{\eta}(P - \hat{c}) - \eta(P - c)) = 0. \tag{44}$$

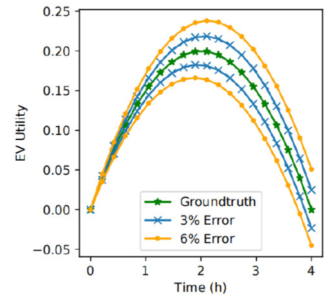
Note that, even without the perfect prediction, as long as the EVCS predictions are unbiased, the overstay fee can cover the opportunity cost on expectation, i.e.,

$$\mathbb{E}_{\eta,c}\big(p^{\max}\Delta t(\hat{\eta}(P - \hat{c}) - \eta(P - c))\big) = 0. \tag{45}$$

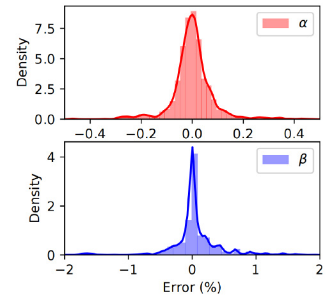
■

D. Utility Function and Demand Estimation Error

Each estimated EV information pattern is represented by a tuple with 4 elements, i.e., $\{\alpha, \beta, \sigma, D\}$, where α and β are the parameters to characterize this EV pattern's quadratic temporal utility function. σ and D denote the population proportion and charging demand of this EV pattern, respectively.

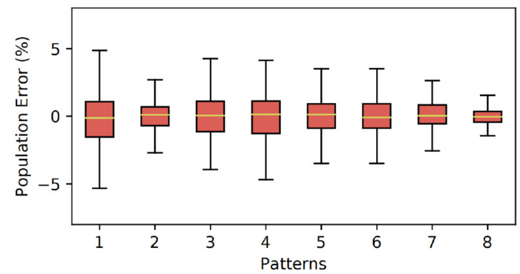


(a) Utility Function Error

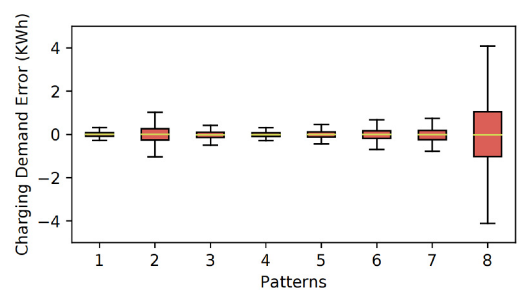


(b) Estimation Errors of α and β

Fig. 18. Estimation Evaluation of α and β.



(a) Population Estimation Error



(b) Demand Estimation Error

Fig. 19. Estimation Evaluation of σ and D.

- Estimation accuracy for α and β: In Fig. 18(a), for a given groundtruth utility function (illustrated by the green line), we visualize the lower and upper bounds of the biased utility function with 3% and 6% input parameter errors (e.g., the income, daily travel distance) . It is clear that, even with 6% error, the difference between the biased utility function (illustrated by the orange lines) and the groundtruth is not very significant. The bias can be further reduced after the information clustering. We conduct 200 estimations with 10% input parameter errors, and the resulting estimation errors of α and β are visualized in Fig. 18(b). We find that the errors of α and β can be well bounded by 0.2% and 1%, which are significantly smaller than the original 10% input parameter error. This implies that the clustering process contains the error propagation.
- Estimation accuracy for σ and D: Fig. 19 illustrates the errors of the population proportions and the charging demand of different patterns. These patterns are ordered in terms of the charging demands ascendingly.

We find that population proportion errors of all patterns are bounded by 5%. Further, the charging demand errors of the first seven patterns are bounded by $1KWh$, and that of the last pattern is bounded by $4KWh$. Since the charging demand of the last pattern is $77KWh$, the relative error is within 5.2%. ■

REFERENCES

[1] S. Wang, S. Bi, Y.-J. A. Zhang, and J. Huang, "Electrical vehicle charging station profit maximization: Admission, pricing, and online scheduling," *IEEE Trans. Sustain. Energy*, vol. 9, no. 4, pp. 1722–1731, Oct. 2018.

[2] Y. Cui, Z. Hu, and X. Duan, "Optimal pricing of public electric vehicle charging stations considering operations of coupled transportation and power systems," *IEEE Trans. Smart Grid*, vol. 12, no. 4, pp. 3278–3288, Jul. 2021.

[3] T. Mao, W.-H. Lau, C. Shum, H. S.-H. Chung, K.-F. Tsang, and N. C.-F. Tse, "A regulation policy of EV discharging price for demand scheduling," *IEEE Trans. Power Syst.*, vol. 33, no. 2, pp. 1275–1288, Mar. 2018.

[4] C. Luo, Y.-F. Huang, and V. Gupta, "Stochastic dynamic pricing for EV charging stations with renewable integration and energy storage," *IEEE Trans. Smart Grid*, vol. 9, no. 2, pp. 1494–1505, Mar. 2018.

[5] J. Soares, M. A. F. Ghazvini, N. Borges, and Z. Vale, "Dynamic electricity pricing for electric vehicles using stochastic programming," *Energy*, vol. 122, pp. 111–127, Mar. 2017. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0360544216319132

[6] C. Lu, Z. Wang, and C. Wu, "Storage-aided service surcharge design for EV charging stations," in *Proc. 60th IEEE Conf. Decis. Control (CDC)*, 2021, pp. 5653–5658. [Online]. Available: https://doi.org/10.1109/CDC45484.2021.9683047

[7] C. Lu, J. Wu, and C. Wu, "Privacy-preserving decentralized price coordination for EV charging stations," *Electr. Power Syst. Res.*, vol. 212, Nov. 2022, Art. no. 108355. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0378779622005259

[8] D. Qiu, Y. Ye, D. Papadaskalopoulos, and G. Strbac, "A deep reinforcement learning method for pricing electric vehicles with discrete charging levels," *IEEE Trans. Ind. Appl.*, vol. 56, no. 5, pp. 5901–5912, Sep./Oct. 2020.

[9] T. Qian, C. Shao, X. Li, X. Wang, Z. Chen, and M. Shahidehpour, "Multi-agent deep reinforcement learning method for EV charging station game," *IEEE Trans. Power Syst.*, vol. 37, no. 3, pp. 1682–1694, May 2022.

[10] J. Tan and L. Wang, "Enabling reliability-differentiated service in residential distribution networks with PHEVs: A hierarchical game approach," *IEEE Trans. Smart Grid*, vol. 7, no. 2, pp. 684–694, Mar. 2016.

[11] A. Moradipari and M. Alizadeh, "Pricing and routing mechanisms for differentiated services in an electric vehicle public charging station network," *IEEE Trans. Smart Grid*, vol. 11, no. 2, pp. 1489–1499, Mar. 2020.

[12] E. Bitar and S. Low, "Deadline differentiated pricing of deferrable electric power service," in *Proc. 51th IEEE Conf. Decis. Control*, 2012, pp. 4991–4997. [Online]. Available: https://doi.org/10.1109/CDC.2012.6425944

[13] E. Bitar and Y. Xu, "Deadline differentiated pricing of deferrable electric loads," *IEEE Trans. Smart Grid*, vol. 8, no. 1, pp. 13–25, Jan. 2017.

[14] A. Ghosh and V. Aggarwal, "Control of charging of electric vehicles through menu-based pricing," *IEEE Trans. Smart Grid*, vol. 9, no. 6, pp. 5918–5929, Nov. 2018.

[15] A. Ghosh and V. Aggarwal, "Menu-based pricing for charging of electric vehicles with vehicle-to-grid service," *IEEE Trans. Veh. Technol.*, vol. 67, no. 11, pp. 10268–10280, Nov. 2018.

[16] F. Salah and C. M. Flath, "Deadline differentiated pricing in practice: Marketing EV charging in car parks," *Comput. Sci. Res. Develop.*, vol. 31, nos. 1–2, pp. 33–40, 2016.

[17] T. Zeng, S. Bae, B. Travacca, and S. Moura, "Inducing human behavior to maximize operation performance at PEV charging station," *IEEE Trans. Smart Grid*, vol. 12, no. 4, pp. 3353–3363, Jul. 2021.

[18] A. Holder, *Parametric LP Analysis*. Hoboken, NJ, USA: Wiley, 2010.

[19] H. R. Varian, "Price discrimination," in *Handbook of Industrial Organization*, vol. 1. New York, NY, USA: North-Holland, 1989, pp. 597–654.

[20] L. Gan, U. Topcu, and S. H. Low, "Optimal decentralized protocol for electric vehicle charging," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 940–951, May 2013.

[21] A. H. Milaghardan, R. A. Abbaspour, and C. Claramunt, "A spatio-temporal entropy-based framework for the detection of trajectories similarity," *Entropy*, vol. 20, no. 7, p. 490, 2018.

[22] "Gurobi Solver 9.5.0." Gurobi. [Online]. Available: https://www.gurobi.com (Accessed: Dec. 20, 2021).

[23] J. Cooper, R. Fox, J. Loeprick, and K. Mohindra, *Transfer Pricing and Developing Economies: A Handbook for Policy Makers and Practitioners*. Washington, DC, USA: World Bank Publ., 2017.

[24] S. Çolak, A. Lima, and M. C. González, "Understanding congested travel in urban areas," *Nat. Commun.*, vol. 7, no. 1, pp. 1–8, 2016.

[25] "ChargePoint Charging Session Data." ChargePoint. 2013. [Online]. Available: https://www.chargepoint.com (Accessed: Oct. 15, 2021).

[26] "California Air Resources Board Clean Vehicle Rebate Project, Rebate Statistics." Center for Sustainable Energy. 2017. [Online]. Available: https://cleanvehiclerebate.org/rebate-statistics (Accessed: Apr. 5, 2017).

[27] "Census Data." United States Census Bureau. [Online]. Available: https://www.census.gov/data.html (Accessed: Oct. 15, 2016).

[28] S. Jiang, Y. Yang, S. Gupta, D. Veneziano, S. Athavale, and M. C. González, "The TimeGeo modeling framework for urban mobility without travel surveys," *Proc. Nat. Acad. Sci.*, vol. 113, no. 37, pp. E5370–E5378, 2016.

[29] Y. Xu, S. Çolak, E. C. Kara, S. J. Moura, and M. C. González, "Planning for electric vehicle needs by coupling charging profiles with urban mobility," *Nat. Energy*, vol. 3, no. 6, pp. 484–493, 2018.

[30] "California Load Data." CAISO. [Online]. Available: https://www.caiso.com/TodaysOutlook/ (Accessed: Apr. 26, 2022).

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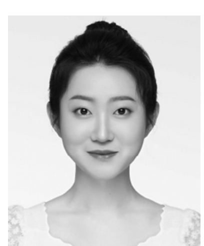

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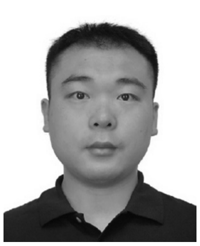

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